

# A Critical Examination of the Calculation of $\Delta H_f^\circ$ Values from *ab Initio* Energies and from Steric Energies Obtained by Molecular Mechanics

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Calculation of enthalpies of formation from steric energies obtained by molecular mechanics is usually based on the equation  $\Delta H_f - SM = \sum n_i a_i + SE$  or a variant; SE is the "steric energy" given by a molecular mechanics calculation. A similar equation has been used for *ab initio* calculations; SE would be replaced by the *ab initio* energy. There are four serious limitations in interpreting the calculated  $\Delta H_f^\circ$  values: (1) Differences in calculated  $\Delta H_f^\circ$  values obtained with different force fields (or basis sets) arise from differences in calibration sets as well as from differences in the force fields (or basis sets) themselves. (2) Any change in the parameters of a force field that lead to revised SE values for a given set of molecules will necessitate recalibration of the  $a_i$  values if calculated  $\Delta H_f^\circ$  values are to be valid. Owing to the large sizes of calibration sets customarily used and, usually, to lack of documentation, recalibration is impractical or impossible and is almost never done. (3) Calculation of  $\Delta H_f^\circ$  requires SM, a correction for the energy contributed by conformers other than the global minimum. The SM (or equivalent) values used in a given study are not generally published. (4) Calibration procedures distribute errors among all compounds of a calibration set and hence diminish or conceal important trends among errors. These several limitations reduce the significance of comparisons among  $\Delta H_f^\circ$  values calculated in different laboratories or within the same laboratory using different force fields. The limitations can be overcome by separating the calculation of  $\Delta H_f^\circ$  into two parts, a larger part (usually), the formal group enthalpy, which is independent of force field or basis set and a smaller part, the formal steric enthalpy (FSE), which is derived through application of the force field or basis set. Both parts are defined in terms of standard reference molecules. The method may be characterized as an extended type of isodesmic calculation. The FSE method has several advantages. First is that calibrations are based on minimal sets of clearly defined standard molecules. This feature facilitates both portability and ease of calibration and recalibration. Second, there is significant cancellation of errors arising from limitations of force field or basis set, an advantage inherent in isodesmic methods. Third is that FSE values provide a tool for making direct comparisons of performances of force fields or of basis sets. Since FSE values are defined in terms of defined standard molecules, every method of calculation must in principle give the identical FSE value for a given molecule. Moreover, useful comparisons of differences of  $\Delta H_f^\circ$  values are possible even though SM values or experimental  $\Delta H_f^\circ$  data are lacking. In the present study the FSE method has been used to evaluate the performance of several basis sets in calculation of FSE values and of  $\Delta H_f^\circ$  values of alkanes and cycloalkanes. The study has turned up an unresolved inconsistency in the performance of the basis sets between acyclic and cyclic alkanes. A similar inconsistency had been noted previously with some force fields. An analysis is presented of the validity of estimating zero point energies and  $\Delta H(0-298)$  values as sums of increments. Results obtained with alkanes are of special significance since most molecules consist primarily of hydrocarbon subunits.

## Introduction

Theoretical calculation of energies of molecules by molecular mechanics, by semiempirical molecular orbital methods, and by *ab initio* methods makes use of a wide variety of force fields, of empirical calculations, and of basis sets and electron correlation treatments. As illustrated below, most current methods for converting calculated energies into  $\Delta H_f^\circ$  values or their equivalents are more or less ambiguous. There have long been concerns about the validity of calculated energies. How can we decide which of several programs or force fields give reliable energies? On the basis of what evidence can we trust results obtained with a modified force field? More generally, how can an author provide evidence of the validity of his or her calculations? A related issue is arising as *ab initio* methods become a routine method of

calculating energies. How should *ab initio* energies be converted to enthalpies? A further consideration is that all methods of theoretical calculation of  $\Delta H_f^\circ$  values involve compromises between accuracy and cost. It is important to have reliable procedures for evaluating the performance of whatever method has been chosen.

In this study I propose a general solution to these several problems, one applicable to the types of compounds for which group increment calculations are applicable. It involves use of the FSE (formal steric enthalpy) procedure for converting energies to enthalpies of formation. The FSE procedure is an implementation of isodesmic calculations; it uses published sets of reference standards. Since every force field and every basis set must in principle yield the identical FSE value for a given molecule (in terms of some given set of standards), comparisons of FSE values provide a direct and objective comparison of the performance of different force fields or basis sets, a comparison that is free of the extraneous

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factors inherent in the use of eq 2 below. In this study I use the FSE procedure to treat *ab initio* energies of representative acyclic and cyclic alkanes in order to compare the effects of choice of basis set and of alternative methods of treating zero point energies and  $\Delta H(0-298)$  quantities on the derived  $\Delta H_f^\circ$  values.

### Methods for Converting Energies to Estimates of $\Delta H_f^\circ$

The ability to convert energies to enthalpies of formation depends on the property of additivity of group increments. Empirical additivity treatments have a long history, summarized admirably by Cox and Pilcher.<sup>1</sup> The most widely used sets of empirical group increments are those derived by Benson,<sup>2-4</sup> these are used in eq 1. The

$$\Delta H_f = \sum n_i b_i + \text{steric corrections} \quad (1)$$

enthalpy of formation of octane, for example, is the sum of  $2b[\text{C}(\text{C})(\text{H})_3] + 6b[\text{C}(\text{C})_2(\text{H})_2]$ ; the steric correction is zero for octane.

Equation 2, a modification of eq 1, has long been used to convert steric energies calculated by molecular mechanics into enthalpies of formation.<sup>5,6</sup> The sum of increments  $\sum n_i a_i$  plus the steric energy SE on the right hand side of equation 2 gives the calculated enthalpy of formation of a hypothetical substance consisting entirely of a single conformer. This is to be the conformer of lowest energy, the global minimum. The left hand side is the enthalpy of formation of the actual substance minus SM (statistical mechanical correction), the contribution to the enthalpy due to conformers having energies higher than that of the conformer of lowest energy. The left hand side is, therefore, also the enthalpy of formation of the hypothetical single conformer of lowest energy. For butane SM is  $\sim 0.27$  kcal/mol, and for octane it is  $\sim 1.12$ .<sup>7-12</sup> SE is the quantity obtained in a molecular mechanics calculation.

$$\Delta H_f - \text{SM} = \sum n_i a_i + \text{SE} \quad (2)$$

Some workers use variants of eq 2; MM2 and MM3 use bond energies plus group corrections in place of group increments.<sup>12-15</sup> Wiberg used a different variant of eq 2 to convert *ab initio* energies into enthalpies of formation.<sup>16</sup>

Equation 2 has four practical limitations. One is that calculated  $\Delta H_f^\circ$  values are dependent on the composition of the calibration set used to derive the  $a_i$  values. An illustration of how large this dependence can be is given below. A consequence of using eq 2 is, therefore, that differences among  $\Delta H_f^\circ$  values derived with different force fields or basis sets may arise from differences of calibration sets as well as from inherent differences of the force fields or basis sets.

A second limitation of eq 2 is rigidity; any change in force field that alters the SE values requires recalibration of the  $a_i$  values if the derived enthalpies are to be correct. Recalibration using an unwieldy calibration set is time consuming at best and is usually quite impossible since the composition of the set used in deriving the original  $a_i$  values is not usually published. This essential step is generally omitted.

A third limitation of eq 2 is that  $\Delta H_f^\circ$  values from different laboratories may be based on different choices of SM values.  $\text{SM} = \sum e_i f_i$ , where  $f_i$  is the Boltzmann fraction of conformers having energy  $e_i$  higher than the energy of the global minimum. The values used are seldom published. SM (POP) values for use with the Allinger MM2 force field have not been published; back calculation from the "strain energies" of Table 3.5 of ref 13 indicates that a value of about 0.30–0.34 kcal/mol was used per methylene group in calculating the  $\Delta H_f^\circ$  data of Table 3.1. On the basis of the 0.30 increment, the resultant POP (SM) values of *n*-alkanes are typically about 30% higher and of isoalkanes about 80% higher than obtained from the defining equation.

Estimation of SM values for acyclic alkanes with methyl branching is relatively straightforward<sup>7-9,11,17-19</sup> since incremental algorithms are applicable. But deriving SM values for complex alkanes or for cycloalkanes having large rings may pose considerable difficulty due to the need to locate large numbers of conformers.<sup>20,21</sup> Assignment of SM values to molecules having functional groups has been insufficiently explored.

A fourth limitation is that the calibration process tends to distribute errors among the entire set of compounds. This characteristic may conceal important trends in the data.

Returning now to the dependence of calculated  $\Delta H_f^\circ$  values on the calibration set employed, the difficulties may be illustrated by examining two published treatments of a set of *ab initio* energies reported by Wiberg<sup>16</sup> for several alkanes and cycloalkanes. Wiberg derived the four alkane increments by fitting a form of eq 2 to most of the data reported. Allinger et al.<sup>22</sup> performed an independent calibration of increments using a subset of the Wiberg data. Calculated  $\Delta H_f^\circ$  values obtained with these two different calibrations show many differences that exceed 1 kcal/mol and some that exceed 3 kcal/mol. This example illustrates the sensitivity of calculations to the makeup of the calibration set and the importance of using defined sets of standards, especially if the calibration set is small, as is apt to be the case for *ab initio* data.

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A way around the shortcomings of eq 2 is to define the two components of  $\Delta H_f^\circ$  differently, eq 3.<sup>7,11</sup> In eq 3 the

$$\Delta H_f - \text{SM} = \sum n_i c_i + \text{FSE} \quad (3)$$

$\sum n_i c_i$  term, the formal group enthalpy, is independent of force field or basis set; the sum will have the same value for all isomeric compounds having the same set of structural units. The formal steric enthalpy (FSE) term represents the differences that arise from angle "strain", nonbonded repulsions, and so on. FSE is the component of  $\Delta H_f^\circ$  that is the appropriate target of calculation. Both the  $\sum n_i c_i$  and the  $n_i d_i$  terms are defined in terms of the same sets of standard molecules as described below.

The FSE value is obtained from the calculated energy of a molecule by use of eq 4 or 5.<sup>7,9,10,12</sup> SE (steric energy)

$$\text{FSE} = \text{SE} - \sum n_i d_i \quad (4)$$

$$\text{FSE} = 627.5(E_{\text{AI}} - \sum n_i d_{\text{AI}_i}) \quad (5)$$

is the energy quantity obtained in a molecular mechanics calculation.  $E_{\text{AI}}$  is an *ab initio* energy in hartrees/molecule. The 627.5 converts to kcal/mol. Note that  $d_i = c_i - a_i$ .

A separate  $d_i$  conversion element and  $c_i$  increment is required for each structural unit in the compound. Any alkane or cycloalkane may be constructed from just four structural units,  $[\text{C}(\text{C})(\text{H})_3]$ ,  $[\text{C}(\text{C})_2\text{H}_2]$ ,  $[\text{C}(\text{C})_3(\text{H})]$ , and  $[\text{C}(\text{C})_4]$ . (The notation is an adaptation of the Benson notation.<sup>3</sup>)

A published set of standard molecules for defining FSE values for acyclic alkanes and cycloalkanes consists of butane (0.00), octane (0.00), 2-methylbutane (0.70), and 2,2-dimethylbutane (1.40). Assigned FSE values are given in parentheses. These molecules were chosen so as to be as representative as possible of the total set of acyclic alkanes and cycloalkanes. These standards have been used in eq 3 along with experimental enthalpies of formation to obtain  $c_i$  values. The same standards have been used with eqs 4 and 5 to derive the  $d_i$  conversion factors appropriate to a given force field or basis set.

If functional groups are present, additional standards are needed. To treat all possible alcohols and ethers requires 12 additional standards, for example. However, just one standard molecule is needed to represent any given structural unit. Standard molecules along with assigned FSE values have been defined for a few classes of molecules. Sets of  $c_i$  increments and experimental FSE values have been published for hydrocarbons, alcohols, ethers, carbonyl compounds, acids, and esters.<sup>11,17-19</sup>

It is to be emphasized that for alkanes calculation of the  $d_i$  conversion terms requires exactly four determinations of SE or of  $E_{\text{AI}}$ , that is, one for each of the four standards. To calibrate or to recalibrate an  $a_i$  set would require calculations for all of the 30 or so alkanes typically used to specify their values. Moreover, the standards and their FSE values are precisely defined. This means that all research groups have access to exactly the same standards. In contrast, calibration of  $a_i$  sets is subject to arbitrary selections of numerical values, to arbitrary choices of the composition of calibration sets, and to serious uncertainties of SM data.

The number of standards required obviously goes up markedly as structural units are added to the list. While there is no "free lunch," calibration and recalibration of

the  $d_i$  conversion factors is practical and is accurately reproducible in all laboratories.

As discussed below, the FSE approach makes it possible in some cases to calculate valid estimates of differences of  $\Delta H_f^\circ$  values even though no experimental data are available, and even if some  $c_i$  and  $d_i$  values are missing.

In the Appendix I show that the FSE formalism is a compact way of performing isodesmic calculations. Application of eq 4 or 5 assures that the target molecule and the mix of standards have identical sets of structural units. Isodesmic reactions have often been used to convert *ab initio* energies to relative enthalpies of formation and enthalpies of reaction.<sup>23-27</sup> Isodesmic calculations tend to cancel out shortcomings of a force field or of a basis set since the limitations may be expected to be similar among target molecules and the standards.

The expected cancellation often works out well in practice. As an example, 6-31G\* is a moderately expensive basis set, but errors of relative energies of atomization of alkanes are still rather large. Thus, calculation of  $\Delta H_f^\circ(\text{octane}) - \Delta H_f^\circ(2,2,3,3\text{-tetramethylbutane})$  as a difference of the *ab initio* energies gives  $-10.1$  kcal/mol. The experimental difference is  $+4.12$  kcal/mol ( $-49.84 + 53.96$ ).<sup>28,29</sup> For these two molecules the 6-31G\* basis set shows an error of more than 14 kcal/mol in what amounts to an estimate of the difference of energies of atomization. (Correction for differences in zero point energies is insignificant.)

The FSE value for octane is 0 (defined) and the experimental FSE value for 2,2,3,3-tetramethylbutane is 6.68.<sup>11</sup> The calculated values are 0 (defined) and 8.60. The corresponding  $\Delta H_f^\circ$  values calculated by eq 3 are  $-49.84$  for octane (calibration standard) and  $-52.03$  for tetramethylbutane. On the basis of either differences of FSE values or equivalently on differences of  $\Delta H_f^\circ$  derived with eq 3, the error is 1.9 kcal/mol instead of 14.

## FSE and $\Delta H_f^\circ$ Values from *ab Initio* Energies

**Comparison of the Characteristics of Different Basis Sets.** The conversion factors  $d_{\text{AI}}$  summarized in Table 4 were derived by substituting the respective FSE and *ab initio* energies (Table 3) of the four standards into eq 5. FSE values derived with eq 5 using appropriate conversion factors are shown in Table 1, and the corresponding  $\Delta H_f^\circ$  data are shown in Table 2. Calculations based on eq 5 assume that group additivity applies to the zero point energies (ZPE's) and to the  $\Delta H(0-298)$  terms as well as to the enthalpy of formation of the frozen molecule at 0 K. All three quantities are subsumed in eq 5. Previous studies based on eq 2 or on direct application of isodesmic calculations likewise have included ZPE and  $\Delta H(0-298)$  terms in the calibration

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Table 1. Primary FSE Values of Alkanes and Cycloalkanes from *ab Initio* Energies (kcal/mol)

compd		3-21G FSE	4-31G FSE	6-31G* FSE	6-31G** FSE	exptl FSE <sup>a</sup>
ethane		0.19	0.15	0.17	0.15	0.02
cyclopropane		35.20	30.92	28.37	27.63	28.15
propane	C <sub>2v</sub>	0.04	0.05	0.04		0.29
bicyclo[1.1.0]butane		85.95	79.18	67.15		66.70
cyclobutane		28.30	27.63	26.19		27.17
2-methylpropane		-0.23	-0.37	-0.45		0.25
butane	C <sub>2h</sub>	0.00	0.00	0.00	0.00	0.01 <sup>b</sup>
butane	gauche	0.77		0.95	0.95	
bicyclo[2.1.0]pentane		64.57	60.32	54.28		57.66
bicyclo[1.1.1]pentane			74.34	67.64		66.81
spiropentane		76.73	65.13	59.68		65.07
cyclopentane		4.42	9.14			7.38
2,2-dimethylpropane		-0.80	-1.03	-1.19		0.43
2-methylbutane	C <sub>1</sub>	0.70	0.70	0.70	0.70	0.58 <sup>b</sup>
2-methylbutane	C <sub>s</sub>	1.41		1.62		
pentane	C <sub>2v</sub>	0.01	0.05	0.00		-0.02
pentane	gauche	0.78		1.01		
pentane	g <sup>+</sup> g <sup>+</sup>	1.51		1.88		
pentane	g <sup>+</sup> g <sup>-</sup> C <sub>s</sub>	4.04		4.11		
pentane	g <sup>+</sup> g <sup>-</sup> C <sub>1</sub>	3.69		3.88		
bicyclo[2.1.1]hexane			38.64			
bicyclo[2.2.0]hexane	cis	54.78	54.04			55.00
bicyclo[2.2.0]hexane	trans		105.53			
cyclohexane		-1.32	-0.34	0.10		1.42
2,2-dimethylbutane		1.40	1.40	1.40	1.40	1.09 <sup>b</sup>
2,3-dimethylbutane	C <sub>2h</sub>	2.64	2.72	2.75		2.10
2,3-dimethylbutane	C <sub>2</sub>	2.67		2.66		2.10
hexane	g(3-4)	0.78		1.05		
hexane	g(2-3)	0.77		1.01		
hexane	g <sup>+</sup> tg <sup>+</sup>	1.63		2.10		
hexane	C <sub>2h</sub>	0.00		0.00		0.01
bicyclo[2.2.1]heptane		9.04	12.16	13.61		17.13
methylcyclohexane	ax	0.17				0.98
methylcyclohexane	eq	-1.74				0.98
heptane	C <sub>2v</sub>	0.00		0.00		0.02
cubane		154.30	158.80	153.09		166.75
bicyclo[2.2.2]octane		4.70	7.86	8.77		11.74
2,2,3,3-tetramethylbutane		8.01	8.49	8.58	8.60	6.68
octane	C <sub>2h</sub>	0.00	0.00	0.00	0.00	-0.01 <sup>b</sup>
octane	g(4-5)	0.77		1.05		
adamantane				0.28		7.74

<sup>a</sup> Reference 11. <sup>b</sup> Standard molecule; the "calculated" FSE values of 0.00, 0.70, and 1.40 are those defined for standards.

sets.<sup>16,22</sup> I evaluate alternative treatments of the ZPE and  $\Delta H(0-298)$  terms below.

The data in Table 1 show that FSE values derived from the 6-31G\* basis set by use of eq 5 reproduce experimental FSE values and, accordingly, through equation 3, the  $\Delta H_f$  values of acyclic alkanes and monocycloalkanes, within about 1 kcal/mol. Results with spiropentane, cubane, and adamantane show larger errors. Since  $\Delta H_f^\circ$  values are calculated from the FSE values of Table 1, they show an identical error pattern.

Direct comparisons are valid among the FSE values obtained with the 6-31G\* basis set and those obtained with other basis sets. For the somewhat limited set of acyclic alkanes for which 3-21G data are available, it appears that the much less expensive 3-21G set gives results of comparable quality to those obtained with use of 6-31G\*. Results with cyclic compounds are sensitive to the basis set. The FSE values, and accordingly the  $\Delta H_f$  values, decrease markedly for most cyclic alkanes on going from 3-21G to 4-21G to 6-31G\*. For cyclic compounds the RHF 3-21G and 4-31G basis sets are clearly inadequate, and it will be shown below that the relatively good agreement found between experimental values and values calculated by eqs 5 and 3 for some cycloalkanes is somewhat illusory.

A previous application of the FSE analysis to the MM2 force field shows that it also treats acyclic alkanes and cycloalkanes inconsistently.<sup>12</sup>

### What about Zero Point Energies and the $\Delta H(0-298)$ Terms?

Since the FSE values of the standards pertain to molecules at 298 K (gas phase), the  $dAI_i$  conversion factors of eq 5 also pertain to compounds at 298 K and, therefore, automatically include zero point energies and  $\Delta H(0-298)$  energies appropriate to the standard molecules. As an aside, some workers state that all comparisons should be made at 0 K. If we are interested in enthalpy data at 298 K, it does not matter numerically whether data are computed for 0 or for 298 K. The same corrections are required for making the conversions in either direction.

In Table 5 I address the question as to whether zero point energies can be represented adequately as sums of group increments. The upper half of the table lists data for acyclic alkanes. The second and third columns list ZPE values reported by Pitzer and by Cottrell. Data in the remaining columns show ZPE values calculated from RHF 6-31G\* frequencies scaled by 90% and by 91% and data from 3-21G frequencies scaled by 90%. For all acyclic molecules the group increments in the middle of the table reproduce listed ZPE values within 0.4 kcal/mol or better. It is well known that frequencies derived by use of the 3-21G basis set are often as good as those obtained with larger basis sets.<sup>25</sup> It may be noted that ZPE values for the larger alkanes are better reproduced

Table 2.  $\Delta H_f^\circ$  Values of Alkanes and Cycloalkanes from Primary FSE Values of Table 1 (kcal/mol)

compd		3-21G $\Delta H_f^\circ$	4-31G $\Delta H_f^\circ$	6-31G* $\Delta H_f^\circ$	6-31G** $\Delta H_f^\circ$	exptl <sup>a</sup> $\Delta H_f^\circ$
ethane		-19.88	-19.92	-19.90	-19.92	-20.06
cyclopropane		19.76	15.48	12.93	12.19	12.71
propane		-25.17	-25.16	-25.17		-24.92
bicyclo[1.1.0]butane		71.14	64.37	52.34		51.89
cyclobutane		7.71	7.04	5.60		6.58
2-methylpropane		-32.59	-32.73	-32.81		-32.11
butane	$C_{2h}$	-30.09	-30.09	-30.09	-30.09	-30.09 <sup>b</sup>
butane	gauche	-29.32		-29.14	-29.14	
bicyclo[2.1.0]pentane		44.61	40.36	34.32		37.70 <sup>c</sup>
bicyclo[1.1.1]pentane			54.38	47.68	46.85	
spiropentane		55.93	44.33	38.88		44.26
cyclopentane		-21.32	-16.60			-18.36
2,2-dimethylpropane		-41.15	-41.38	-41.54		-39.93
2-methylbutane	$C_1$	-36.71	-36.71	-36.71	-36.71	-36.83 <sup>b</sup>
2-methylbutane	$C_s$	-36.00		-35.79		
pentane	$C_{2v}$	-35.03	-34.99	-35.04		-35.06
pentane	gauche	-34.26		-34.03		
pentane	$g^+g^+$	-33.53		-33.16		
pentane	$g^+g^- C_s$	-31.00		-30.93		
pentane	$g^+g^- C_1$	-31.35		-31.16		
bicyclo[2.1.1]hexane			13.54			
bicyclo[2.2.0]hexane	cis	29.68	28.94			29.90
bicyclo[2.2.0]hexane	trans		80.43			
cyclohexane		-32.20	-31.22	-30.78		-29.46
2,2-dimethylbutane		-44.10	-44.10	-44.10	-44.10	-44.42 <sup>b</sup>
2,3-dimethylbutane	$C_{2h}$	-42.01	-41.93	-41.90		-42.55
2,3-dimethylbutane	$C_2$	-41.98		-41.99		-42.55
hexane	$g(3-4)$	-39.18		-38.91		
hexane	$g(2-3)$	-39.19		-38.95		
hexane	$g^+tg^+$	-38.33		-37.86		
hexane	$C_{2h}$	-39.96		-39.96		-39.95
bicyclo[2.2.1]heptane		-21.21	-18.09	-16.64		-13.12
methylcyclohexane	ax	-37.78				
methylcyclohexane	eq	-39.70				-36.98
heptane	$C_{2v}$	-44.89		-44.89		-44.87
cubane		136.24	140.74	135.03		148.69
bicyclo[2.2.2]octane		-30.70	-27.54	-26.63		-23.66
2,2,3,3-tetramethylbutane		-52.62	-52.14	-52.05	-52.03	-53.96
octane	$C_{2h}$	-49.83	-49.83	-49.83	-49.83	-49.84 <sup>b</sup>
octane	$g(4-5)$	-49.06		-48.78		
adamantane				-39.63		-32.17

<sup>a</sup> References 1, 28, and 29. <sup>b</sup> Standard. <sup>c</sup> Reference 16.

by a larger scaling factor than 90%; starting with pentane a factor of about 91.6% is better with the 6-31G\* basis set. Since ZPE values scale linearly, the values that correspond to any desired factor are readily obtainable from data derived with another factor by using an appropriate ratio.

Data in the lower part of Table 5 show that ZPE values for cycloalkanes are less well represented as sums of group increments. The summation of increments yields ZPE values that are larger than the reported values by from 2 to 4 kcal/mol or more. Direct estimates of ZPE values may, of course, be obtained from calculated frequencies; with a scaling factor of 90% the calculated ZPE values, based on 6-31G\*\* frequencies, of cyclopropane, cyclobutane, and cyclohexane differ by from -2 to +1.5 kcal/mol from those reported by Cottrell and by Pitzer.

In Table 6 I examine the parallel question of representing  $\Delta H(0-298)$  values.  $\Delta H(0-298)$  is the change in  $\Delta H_f^\circ$  for a gas on going from 0 to 298 K. Like Table 5, the upper part gives data for acyclic alkanes. The experimental data come from the API-Carnegie Press tables.<sup>30</sup> The experimental values are closely reproduced

by those calculated from frequencies obtained from 6-31G\* values scaled by 90%. The conclusions are similar to those pertaining to ZPE values;  $\Delta H(0-298)$  values from group increments represent those for acyclic molecules accurately but are too high by variable amounts, up to more than 2 kcal/mol for the available cycloalkanes.

Table 7 summarizes the joint results of representing the sum of ZPE and the  $\Delta H(0-298)$  in terms of increments derived from the standard set of molecules, those that define FSE values of acyclic alkanes and cycloalkanes. The differences are variable and may exceed 4 kcal/mol. Noting again that the  $dAI_i$  conversion terms include both ZPE and  $\Delta H(0-298)$  as increments, this procedure treats acyclic alkanes appropriately, as has been demonstrated from the results presented in Tables 5 and 6. However, the incremental representation inherent in eq 5 is less suitable for the cycloalkanes for which data are available.

Equation 6 is an expanded form of eq 5 that treats zero point energy and the  $\Delta H(0-298)$  energy separately.

$$\text{FSE} = 627.5(E_{AI} + \text{ZPE}/627.5 + \Delta H(0-298)/627.5 - \sum n_i dAI_i') \quad (6)$$

Table 8 presents the revised  $dAI_i'$  increments and the FSE values calculated for the available cycloalkanes. The

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Table 3. *Ab Initio* Energies of Alkanes and Cycloalkanes (Hartrees/Molecule)

compd		3-21G energy	4-31G energy	6-31G* energy	6-31G** energy	refs <sup>a</sup>
ethane		78.793 950	79.115 933	79.228 755	79.238 271	16, 39, 39, 39
cyclopropane		116.401 210	116.883 858	117.058 865	117.069 127	16, 39, 39, 39
propane	<i>C</i> <sub>2v</sub>	117.613 300	118.093 810	118.263 650		16, 40, 16
bicyclo[1.1.0]butane		153.986 630	154.624 730	154.871 770		16, 16, 16
cyclobutane		155.231 320	155.866 810	156.097 030		16, 16, 16
2-methylpropane		156.434 460	157.072 590	157.298 960		16, 16, 16
butane	<i>C</i> <sub>2h</sub>	156.432 466	157.071 596	157.298 409	157.313 950	ts, ts, ts
butane	gauche	156.431 244		157.296 895	157.312 440	ts, -, ts, 41
bicyclo[2.1.0]pentane		192.839 810	193.632 490	193.926 966		16, 39, 39
bicyclo[1.1.1]pentane			193.610 156	193.905 681	193.918 816	-, 39, 39, 39
spiropentane		192.821 040	193.624 400	193.917 530		16, 16, 16
cyclopentane		194.088 470	194.873 990			16, 16
2,2-dimethylpropane		195.256 710	196.051 350	196.333 830		16, 16, 16
2-methylbutane	<i>C</i> <sub>1</sub>	195.252 075	196.048 602	196.331 813	196.350 432	ts, ts, ts, ts
2-methylbutane	<i>C</i> <sub>s</sub>	195.250 950		196.330 350		41, -, 41
pentane	<i>C</i> <sub>2v</sub>	195.251 561	196.049 220	196.333 097		ts, 16, ts
pentane	gauche	195.250 320		196.331 500		41, -, 41
pentane	g <sup>+</sup> g <sup>+</sup>	195.249 160		196.330 100		41, -, 41
pentane	g <sup>+</sup> g <sup>-</sup> <i>C</i> <sub>s</sub>	195.245 140		196.326 550		41, -, 41
pentane	g <sup>+</sup> g <sup>-</sup> <i>C</i> <sub>1</sub>	195.245 690		196.326 920		41, -, 41
bicyclo[2.1.1]hexane			232.644 758			-, 39
bicyclo[2.2.0]hexane	cis	231.674 500	232.620 206			16, 39
bicyclo[2.2.0]hexane	trans		232.538 151			-, 39
cyclohexane		232.916 730	233.866 810	234.208 000		41, 16, 16
2,2-dimethylbutane		234.072 301	235.025 182	235.364 379	235.386 088	ts, ts, ts, ts
2,3-dimethylbutane	<i>C</i> <sub>2h</sub>	234.069 710	235.023 498	235.363 060		ts, ts, 41
2,3-dimethylbutane	<i>C</i> <sub>2</sub>	234.069 660		235.363 200		41, -, 41
hexane	g(3-4)	234.069 430		235.366 120		41, -, 41
hexane	g(2-3)	234.069 440		235.366 190		41, -, 41
hexane	g <sup>+</sup> tg <sup>+</sup>	234.068 070		235.364 450		41, -, 41
hexane	<i>C</i> <sub>2h</sub>	234.070 668		235.367 792		ts, -, ts
bicyclo[2.2.1]heptane		270.566 500	271.664 670	272.061 160		16, 16, 16
methylcyclohexane	ax	271.735 060				41, -
methylcyclohexane	eq	271.738 110				41, -
heptane	<i>C</i> <sub>2v</sub>	272.889 774		274.402 484		ts, -, ts
cubane		305.695 680	306.928 890	307.393 620		16, 16, 16
bicyclo[2.2.2]octane		309.392 520	310.649 230	311.103 580		16, 16, 16
2,2,3,3-tetramethylbutane		311.703 836	312.969 694	313.421 136		ts, ts, ts
octane	<i>C</i> <sub>2h</sub>	311.708 880	312.982 438	313.437 179	313.464 828	ts, ts, ts, ts
octane	g(4-5)	311.707 650		313.435 510		41, -, 41
adamantane				388.026 500		-, -, 42

<sup>a</sup> ts = this study.

Table 4. *d*-Conversions for the Basis Sets

	<i>d</i> [C(C)(H) <sub>3</sub> ]	<i>d</i> [C(C)2(H) <sub>2</sub> ]	<i>d</i> [C(C) <sub>3</sub> (H)]	<i>d</i> [C(C) <sub>4</sub> ]
3-21G	-39.397 130 3	-38.819 103 3	-38.242 696 3	-37.666 907 6
4-31G	-39.558 087 5	-38.977 710 5	-38.397 744 5	-37.817 352 6
6-31G*	-39.614 512 8	-39.034 692 3	-38.454 697 8	-37.873 866 6
6-31G**	-39.619 255 5	-39.037 719 5	-38.456 061 5	-37.873 577 6
<i>c</i> -values	-10.033 <sup>a</sup>	-5.147	-2.258	-0.217

<sup>a</sup> This corrects a typographical error in the value reported in the footnote of Table 1 of ref 11; the correct value appears in the other tables of the reference.

residual errors of FSE values of 2.8–4.5 kcal/mol represent limitations of estimation of  $\Delta H_f^\circ$  of cycloalkanes from RHF calculations with the 6-31G\* basis set. Regardless of whether *ab initio* energies from RHF/6-31G\* calculations are converted to FSE values through use of eq 5 or of eq 6, the FSE values of cycloalkanes have larger uncertainties than are desirable.

### Conclusions

The Benson summation of empirical group increments, eq 1, already provides an excellent method of extrapolating known  $\Delta H_f^\circ$  data and is widely used for this purpose.<sup>31–35</sup> The goals of calculation should, therefore,

focus on making valid estimates of  $\Delta H_f^\circ$  for those compounds not amenable to the Benson treatment. These include compounds for which "steric corrections" of eq 1 cannot readily be estimated and compounds for which  $\Delta H_f^\circ$  data are limited or nonexistent.

Most methods of calculation reproduce experimental  $\Delta H_f^\circ$  values of acyclic compounds of the sort that require minimal "steric corrections" in application of eq 1. While that result is obviously necessary, it does not add any capability not already inherent in the very much simpler Benson treatment.

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Table 5. Zero Point Energies

	Pitzer ref 43	Cottrell ref 44	6-31G* 0.90 <sup>a</sup>	6-31G* 0.91 <sup>a</sup>	3-21G 0.90 <sup>a</sup>
methane	26.85	27.10	26.98	27.28	27.10
ethane	45.26	45.20	45.04	45.55	45.21
propane	62.92	63.00	62.45	63.14	62.65
butane	80.62	80.60	79.73	80.61	80.05
pentane	98.32	98.50	96.98	98.06	97.24
hexane	(116.02) <sup>b</sup>	116.20	114.21	115.48	114.49
heptane	(133.72) <sup>b</sup>	(133.95) <sup>b</sup>	131.44	132.90	131.75
octane	(151.42) <sup>b</sup>	(151.70) <sup>b</sup>	148.66	150.31	148.99
2-methylbutane	97.61		96.81	97.88	97.18
2,2-dimethylpropane	96.80				
2,2-dimethylbutane		114.50	113.73	114.99	114.04
2,3-dimethylbutane (C <sub>2h</sub> )			113.94	115.20	114.21
CH <sub>3</sub> increment	22.61	22.60	22.62	22.86	22.72
CH <sub>2</sub> increment	17.70	17.75	17.20	17.43	17.26
CH increment	12.08		11.75	11.88	11.76
C increment	7.17	6.35	6.05	6.12	5.90

	reported values	calcd using Cottrell increments	calcd using 6-31G* (0.90) increments	6-31G* value 0.90 scaling
2-methylpropane	80.3 <sup>c</sup>	79.9	79.6	
cyclopropane	49.1 <sup>d</sup>	53.1	51.6	49.3
cyclobutane	67.3 <sup>d</sup>	70.9	68.8	66.7
cyclopentane	86.0 <sup>d</sup>	88.5	86.0	
cyclohexane	104.5 <sup>d</sup>	106.2	103.2	
methylcyclohexane	120.3 <sup>e</sup>	123.3	120.4	
bicyclo[2.2.1]heptane	105.0 <sup>f</sup>	112.7	109.5	
bicyclo[2.2.2]octane	124.1 <sup>f</sup>	130.4	126.7	
cubane	80.7 <sup>g</sup>	96.6	94.0	
adamantane	148.6 <sup>g</sup>	154.5	150.2	

<sup>a</sup> Factor by which Gaussian frequencies have been multiplied.

<sup>b</sup> Extrapolated. <sup>c</sup> Reference 43. <sup>d</sup> Reference 44. <sup>e</sup> Reference 41. <sup>f</sup> Reference 40. <sup>g</sup> Reference 42.

Table 6.  $\Delta H(298K) - \Delta H(0K)$ 

	ref 30	6-31G* (0.90)
methane	2.40	2.39
ethane	2.86	2.81
propane	3.51	3.49
butane	4.64	4.57
pentane	5.63	5.80
hexane	6.62	6.88
heptane	7.62	7.95
octane	8.81	9.04
2-methylbutane	5.30	5.18
2,2-dimethylbutane	5.91	5.90
CH <sub>3</sub> increment	1.248	1.196
CH <sub>2</sub> increment	1.041	1.111
CH increment	0.515	0.481
C increment	-0.113	-0.005

	ref 30	calcd from increments	6-31G* (0.90)	calcd from increments
2-methylpropane	4.28	4.26		
2,2-dimethylpropane	5.05	4.88		
2,3-dimethylbutane (C <sub>2h</sub> )	5.92	6.02	6.04	5.75
2,2,3,3-tetramethylbutane	7.40	7.26	7.31	7.16
cyclopropane	2.74 <sup>a</sup>	3.12	2.73	3.33
cyclobutane	3.24	4.16	3.25	4.44
cyclopentane	3.60	5.21		
cyclohexane	4.24	6.24		
methylcyclohexane	5.23	7.80		

<sup>a</sup> Reference 44.

The successes of molecular mechanics and of *ab initio* estimates of  $\Delta H_f^\circ$  of compounds requiring larger "steric corrections" tend to be over-rated. In comparing small differences between calculated and experimental  $\Delta H_f^\circ$  values, it is not generally mentioned that the calculated enthalpy component is relatively small; most of the enthalpy occurs in the sum of increments term common

Table 7. ZPE +  $\Delta H(0-298K)$  for Cycloalkanes<sup>a</sup>

	exptl	from acyclic increments	diff
cyclopropane	51.8	56.2	4.4
cyclobutane	70.5	75.1	4.6
cyclopentane	89.6	93.7	4.1
cyclohexane	110.8	112.4	1.6

<sup>a</sup> Data from Tables 5 and 6.

to all molecules. Moreover, the calculations customarily use large numbers of disposable parameters. This has the effect of minimizing individual discrepancies by distributing them over many compounds.

What we should expect of a calculation is that it will reproduce accurately differences of energies of atomization of molecules at 298 K or, equivalently, differences of  $\Delta H_f^\circ$  of single conformers. Such accuracy for *ab initio* energies is beyond present capabilities except for very small molecules. An alternative goal, and one equally useful, is that a calculation should reproduce differences of  $\Delta H_f^\circ$  based on isodesmic comparisons with some set of standard molecules. This may be an achievable goal.

The FSE method provides the requisite isodesmic estimates. It uses no disposable parameters. Comparison of calculated FSE values with experimental FSE values is free of all problems of calibration set size and effects of choices of disposable parameters. The goal of the calculations is clearly defined: calculated FSE values must agree with experimental FSE values subject to the usual caveats about possible limitations of the experimental values. Two methods of calculation must give identical FSE values for a given set of molecules, or else one or both calculations are in error.

I have presented data to show that zero point energies and  $\Delta H(0-298)$  values of acyclic alkanes are adequately representable as summations of increments; this means that eq 5 is strictly valid for these molecules. The analysis also shows that sums of increments are inadequate for cycloalkanes, that eq 6 may be more appropriate. Empirically, for monocycloalkanes, there is significant cancellation of errors that arise from inadequacy of the basis set and from inadequacy of eq 5. The result is that eq 5, though incorrect in principle, gives reasonably correct FSE values for monocycloalkanes. Use of eq 6 requires making relatively expensive frequency calculations for all target molecules; it does not give improved results for monocycloalkanes.

However, in seeking to use eq 6, there is cause for concern in the fact that the 90% scaling factor leads to underestimates of ZPE by about 2 kcal/mol for heptane and octane. It appears that there may be no simple and inexpensive way to get ZPE values of the desired accuracy of, say, 0.5 kcal/mol. Some improvement can perhaps be achieved by adopting an empirical scaling protocol. In contrast to problems with estimating zero point energies to the desired accuracy, there seems to be no difficulty in getting good calculated values of  $\Delta H(0-298)$ .

In many studies it is *differences* of  $\Delta H_f^\circ$  values rather than absolute values that are needed. Between two closely similar molecules there should be substantial cancellation of all types of errors, and the *differences* may have residual errors as small as 0.5 kcal/mol. As an example, the difference of calculated FSE values for bicyclo[2.2.1]heptane and bicyclo[2.2.2]octane is 4.84 kcal/mol and the difference of experimental FSE values is 5.39; the error is 0.55 kcal/mol. On the other hand,

Table 8. Calculation of FSE of Cycloalkanes with ZPE and *H*(0-298) Treated Separately<sup>a</sup>

	AIE	<i>n</i> (CH <sub>2</sub> )	ZPE	<i>H</i> (0-298)	FSE calcd	FSE exptl	Δ
cyclopropane	-117.058 865	3	49.28	2.73	25.33	28.15	2.82
cyclobutane	-156.097 201	4	66.71	3.25	22.64	27.17	4.53
cyclohexane	-234.208 000	6	104.50	4.24	-1.26	1.42	2.68
<i>d</i> [C(C)(H) <sub>3</sub> ]	-39.576 584						
<i>d</i> [C(C) <sub>2</sub> (H) <sub>2</sub> ]	-39.005 449						
<i>d</i> [C(C) <sub>3</sub> (H)]	-38.435 192						
<i>d</i> [C(C) <sub>4</sub> ]	-37.864 179						

<sup>a</sup> AI energies, ZPE's and Δ*H*(0-298) based on RHF 6-31G\* calculations. *Ab initio* energies in hartrees/molecule; other energies in kcal/mol.

calculated individual FSE values for these two compounds are lower than the experimental values by about 3 kcal/mol. (The error between differences of calculated and experimental Δ*H*<sub>f</sub><sup>o</sup> values based on eq 3 is necessarily the same as the difference of FSE values; such cancellations will not necessarily be true of Δ*H*<sub>f</sub><sup>o</sup> values derived with eq 2.)

By taking advantage of cancellations it is possible to calculate certain differences of enthalpies of formation even if one or more *d*<sub>i</sub> conversion values are unknown. For example Δ*H*<sub>f</sub><sup>o</sup>(2,4-di-*tert*-butylpyrrolidine) - Δ*H*<sub>f</sub><sup>o</sup>(2,4-dimethylpyrrolidine) can be calculated from the SE or *ab initio* values using only the alkane-derived *c*<sub>i</sub> and *d*<sub>i</sub> values; the *c*[N(C)<sub>2</sub>(H)] and *d*[N(C)<sub>2</sub>(H)] cancel in the difference.

The FSE method is widely applicable and fast. It took ~20 min with a calculator to derive the *d*<sub>i</sub> conversion values applicable to a set of extended Hückel data<sup>45</sup> and use them to calculate a few FSE values to determine whether Hückel energies might prove useful to estimate Δ*H*<sub>f</sub><sup>o</sup> values. They definitely cannot; errors run to hundreds of kcal/mol. The same calculation with eq 2 would require far longer.

### Summary

The FSE method overcomes most of the limitations that arise from using eq 2 for converting calculated energies to enthalpies. Error trends among FSE values are sharply delineated and arise only from the properties of the force field or the basis set operating on specific molecules.

(1) There are no calibration errors since calibration is not required. FSE values are derived by isodesmic calculations based on defined standards. (2) There need be no recalibration errors if a force field has been modified since revised *d*<sub>i</sub> or *d*AI<sub>i</sub> conversion terms can readily be obtained with relatively few calculations. (3) The SM problem is partly solved. FSE values can be compared directly among calculations with different force fields or basis sets; it is not necessary to convert them to Δ*H*<sub>f</sub><sup>o</sup> values for this purpose. (4) Because FSE values are not based on calibrations, errors for individual molecules are clearly delineated; they are not obscured through distribution over the entire set of data.

In answer to the questions raised about validating calculations made with a specific force field, one possibility is for an author to present FSE values obtained with that force field for a representative selection of molecules. These should be chosen so as to permit adequate comparisons with experimental FSE values and/or with FSE values derived from alternative force fields or basis sets.

A specialized force field derived by manual modification (or by some such procedure as FUDGIT<sup>37</sup>) could be accepted as valid if it gives comparable FSE results for a representative selection. Otherwise, it must be regarded as of limited use.

### Calculations

All calculations were performed on the Cray-Y-MP/432 computer at Florida State University using the Gaussian 90 program.<sup>38</sup> I gratefully acknowledge support by Florida State University through allocation of supercomputer resources.

Geometry optimization and *ab initio* energies were performed with the same basis sets throughout. Gaussian output data were processed on a PC station with help of various programs in AWK and BASIC, with the modeling programs ALCHEMY (Tripos Corp) and PCModel (Serena Software), and by use of MM2(85) to get detailed listings of geometries from cartesian coordinates.

### Selection of Standards for New Classes of Compounds

Suppose that standards are to be chosen for some new class of compounds. The first step is to determine what structural units are necessary to describe all members of the class. The Benson tables<sup>3</sup> may be consulted for this step. For each structural unit the next step is to choose an appropriate molecule that contains the unit; this will constitute one of the standard molecules that define FSE values for the new class. This step is repeated for each structural unit. The selected standard molecules should be as representative as possible of the class; they are isodesmic standards. In assigning FSE values to the new standards, the aim is to make a reasonable approximation of the nonbonded interactions or of other strain. Providing that the calculations are performed consistently, as is essential, the particular FSE values assigned to the new standards do not affect the Δ*H*<sub>f</sub><sup>o</sup> values calculated by eq 3 since they occur identically in both the *c*<sub>i</sub> increments and in the *d*<sub>i</sub> (and *d*AI<sub>i</sub>) conversion terms. Whatever FSE value is assigned to a standard, it will have no effect on the calculated Δ*H*<sub>f</sub><sup>o</sup> values.

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## Appendix

**FSE Calculations Are Based Indirectly on Isodesmic Differences.** For calculated FSE this relationship can be illustrated by a specific example, calculation of FSE for 2,2,3,3-tetramethylbutane (TMB). The relevant standard molecules are 2,2-dimethylbutane (DMB), butane (BUT), and octane (OCT).

Letting  $C_{AI}$  represent the *ab initio* energy in kcal/mol and letting  $d(X)$  be the value of  $dAI(X)$  in kcal, the specific representations of eq 5 are as follows:

$$0 = C_{AI}(\text{BUT}) - 2d(\text{CH}_3) - 2d(\text{CH}_2)$$

$$0 = C_{AI}(\text{OCT}) - 2d(\text{CH}_3) - 6d(\text{CH}_2)$$

$$1.40 = C_{AI}(\text{DMB}) - 4d(\text{CH}_3) - d(\text{CH}_2) - d(\text{C})$$

$$\text{FSE}(\text{TMB})_{\text{calc}} = C_{AI}(\text{TMB}) - 6d(\text{CH}_3) - 2d(\text{C})$$

Solving the first two for  $d(\text{CH}_3)$  and  $d(\text{CH}_2)$  and the third to get  $d(\text{C})$  gives the following:

$$d(\text{CH}_3) = (3/4)C_{AI}(\text{BUT}) - (1/4)C_{AI}(\text{OCT})$$

$$d(\text{CH}_2) = -(1/4)C_{AI}(\text{BUT}) + (1/4)C_{AI}(\text{OCT})$$

$$d(\text{C}) = C_{AI}(\text{DMB}) - (11/4)C_{AI}(\text{BUT}) + (3/4)C_{AI}(\text{OCT}) - 1.40$$

Substituting the  $d(X)$  values into the equation for  $\text{FSE}(\text{TMB})$  gives the following expression in terms of the *ab initio* energies of the standards plus the 2.80 value, which is twice the FSE value assumed for DMB.

$$\text{FSE}(\text{TMB})_{\text{calc}} = C_{AI}(\text{TMB}) + C_{AI}(\text{BUT}) - 2C_{AI}(\text{DMB}) + 2.80$$

In summary,  $\text{FSE}(\text{TMB})_{\text{calc}}$  is the calculated value of the extra component of  $\Delta H_f$  (the "strain") of TMB that arises from the additional crowding of the methyl groups in tetramethylbutane as compared with dimethylbutane.  $\text{FSE}(\text{TMB})_{\text{calc}}$  is based on the difference between the *ab initio* energy for TMB and a sum of conversion factors derived from *ab initio* energies of the reference molecules, corrected for their own FSE values. It is obvious that all *ab initio* values for a given set of calculations must be obtained with the identical protocol (basis set, electron correlation, if any, etc).

A similar derivation of experimental FSE ( $\text{FSE}(\text{TMB})_{\text{exp}}$ ) based on *c*-increments shows parallels with the derivation of  $\text{FSE}(\text{TMB})_{\text{calc}}$ . It should be kept in mind,

however, that *c*-increments are actually based on averages of sets of experimental enthalpy values rather than on the single values used in the illustration. The FSE value and the FSE values assigned to standards cancel out in conversion of FSE into  $\Delta H_f$ .

The applicable specific versions of eq 3 are listed first.

$$\Delta H_f(\text{octane}) - \text{SM}(\text{octane}) = 2c(\text{CH}_3) + 6c(\text{CH}_2) + 0$$

$$\Delta H_f(\text{butane}) - \text{SM}(\text{butane}) = 2c(\text{CH}_3) + 2c(\text{CH}_2) + 0$$

$$\Delta H_f(\text{DMB}) - \text{SM}(\text{DMB}) = 4c(\text{CH}_3) + c(\text{CH}_2) + c(\text{C}) + 1.40$$

$$\Delta H_f(\text{TMB}) - \text{SM}(\text{TMB}) = 6c(\text{CH}_3) + 2c(\text{C}) + \text{FSE}(\text{TMB})_{\text{exp}}$$

As with the *d*-values above, the first three equations may be solved for *c*-values.

$$c(\text{CH}_3) = (3/4)[\Delta H_f(\text{BUT}) - \text{SM}(\text{BUT})] - (1/4)[\Delta H_f(\text{OCT}) - \text{SM}(\text{OCT})]$$

$$c(\text{CH}_2) = -(1/4)[\Delta H_f(\text{BUT}) - \text{SM}(\text{BUT})] + (1/4)[\Delta H_f(\text{OCT}) - \text{SM}(\text{OCT})]$$

$$c(\text{C}) = [\Delta H_f(\text{DMB}) - \text{SM}(\text{DMB})] - (11/4)[\Delta H_f(\text{BUT}) - \text{SM}(\text{BUT})] + (3/4)[\Delta H_f(\text{OCT}) - \text{SM}(\text{OCT})] - 1.40$$

$$\text{FSE}(\text{TMB})_{\text{exp}} = [\Delta H_f(\text{TMB}) - \text{SM}(\text{TMB})] + [\Delta H_f(\text{BUT}) - \text{SM}(\text{BUT})] - 2[\Delta H_f(\text{DMB}) - \text{SM}(\text{DMB})] + 2.80$$

The  $\text{FSE}(\text{TMB})_{\text{exp}}$  is the difference of the  $\Delta H_f$  of TMB minus factors based on the enthalpies of formation of the standard molecules corrected for their assigned FSE values. It is important to note that the FSE values assigned to the standards affect the calculated FSE value and the experimental FSE value identically. The particular assignment of FSE values to standards has absolutely no effect on the relationships between experimental and calculated enthalpies of formation since the assigned FSE values cancel out between eqs 4 and 5. The objective in assigning FSE values to standards is to reflect as well as possible what we mean by "strain."